

1. Consider $I = \int_1^3 x^{3/2} dx$

- (a) Use the FTC to evaluate this integral and record an “exact value” (to 6 decimal places) for I .
- (b) By hand (i.e. using the TI only for function values), compute the middle Riemann sums and trapezoid rule sums that (using 2 and 4 sub-intervals) approximate our integral. Record your answers to 6 decimal places.

Note: We’ll call the four subinterval approximations $M(.5)$ and $T(.5)$, where M means “midpoint approximation” and the the .5 is the size of the subinterval. Since the interval we are using, $[1,3]$, has length 2, a four sub-interval approximation has sub-intervals of length .5.

- (c) If $A(h)$ is an approximation (using a subinterval size of h) for I , we can compute $E_A(h)$, the error for $A(h)$, as $E_A(h) = I - A(h)$. Use your answers to parts (a) and (b) to fill in the appropriate rows showing the calculated errors in your approximations.

subinterval size – h	Midpoint Rule– $M(h)$	Midpoint Error– $E_M(h)$	Trapezoid Rule – $T(h)$	Trapezoid Error– $E_T(h)$	
$\frac{1}{2}$ (4 boxes)					
$\frac{1}{4}$ (8 boxes)					
$\frac{1}{8}$ (16 boxes)					
$\frac{1}{16}$ (32 boxes)					

- 2. Find a substitution function, $u(x)$, and an integrand $g(u)$ such that:

$$\int_1^3 x^{3/2} dx = \int_{-1}^1 g(u) du$$

- 3. Find an antiderivative, $\int \cos(x^2) dx$.

Slides and other material from this talk can be found at

www.cgl.ucsf.edu/home/bic.

Error Analysis Road Map

1. Obtain the error on a single subinterval in terms of h :
 - a. expand $f(x)$ in a Taylor series about the midpoint
 - b. express the integral in terms of h using the series
 - c. express the function values at the endpoints in terms of h using the series
 - d. express the numerical approximation in terms of h using the previous results
 - e. express the error (integral minus approximation) in terms of h
2. Obtain the total error for the entire integral in terms of h :
 - a. add the errors for the subintervals
 - b. argue that certain key expressions that involve h are actually independent of h
 - c. determine the coefficient of the leading term of the series for the error