

## Gamma Function Kwikis (Selections)

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These are examples of exercises given to students, working in groups, over the course of several class days prior to David's talk. You can get this sheet and other material from our presentation by starting at <http://www.cgl.ucsf.edu/home/bic>

1. Let  $f_x(t) = t^{x-1}e^{-t}$ . Graph  $f_x(t)$  on  $[0, 8]$  for  $x$  values of  $0, \frac{1}{2}, 1, 2, 3, 4,$  and  $5$ .

2. Define a function  $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$  for  $x > 0$

(a) Show  $\Gamma(x + 1) = (x)\Gamma(x)$ .

(b) Show  $\Gamma(1) = 1$ .

(c) Compute  $\Gamma(5)$  without a calculator.

3. Recall <sup>1</sup> we define the Gamma function as

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt \text{ for } x > 0 \text{ and you showed}$$
$$\Gamma(x + 1) = (x)\Gamma(x).$$

Use your TI to get a value for  $\Gamma(\frac{1}{2})$ .

4. Evaluate:  $\int \cos^2(\theta) d\theta$

5. Show

$$\int \cos^n(\theta) d\theta = \frac{\cos^{n-1}(\theta) \sin(\theta)}{n} + \frac{n-1}{n} \int \cos^{n-2}(\theta) d\theta$$

Why is this a valuable formula?

6. Use the previous problem for:

$$\int_0^{\pi/2} \cos^n(\theta) d\theta$$

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<sup>1</sup>This exercise was given a few days after the previous one.